

## SUMMARY - MEETING VI BEVATRON RESEARCH CONFERENCE

November 10, 1953

4 PM Auditorium, Bldg. 50

Joseph Lepore: High Energy Nuclear Interaction (Cont.)

GENERAL

The relative probabilities for alternate processes initiated by a nucleon-nucleon collision depend on the dynamics involved and on the volume in phase space accessible to each final state. According to Fermi this last aspect alone may be of decisive importance. Such an assignment would follow were an approximate statistical equilibrium reached.

The volume occupied by the incident nucleons and the associated pion clouds will have very large energy density at the instant of collision. Knowing that the interactions of the pion fields are strong, Fermi assumes that this energy is suddenly distributed among the various degrees of freedom present in the interaction volume in accord with statistical laws. He then computes the statistical probability that a certain number of pions will be created with a given energy distribution. It is then assumed that the energy will rapidly dissolve and that the particles into which the energy has been converted will fly out in all directions. Fermi argues that this approach may give a fairly good approximation to the actual case, since the number of possible states of the given energy is large and the probability of establishing a state to its average statistical weight will be increased by the number of ways by which that state may be attained. The statistical equilibrium assumed above should be qualified to the extent that certain laws such as conservation of charge and conservation of moment must be fulfilled. In addition, only those states easily obtainable from the initial state will reach statistical equilibrium; conservation of the differences of the number of nucleons and the anti-nucleons is implied.

FURTHER DISCUSSION OF THE FERMI METHOD

Return to the calculation of the interaction volume in which the two very energetic nucleons approach in the center of gravity system. The surrounding pion clouds will be Lorentz contracted and the interacting volume will be reduced correspondingly to:

$$\Omega = \Omega_0 \frac{Mc^2}{E} \quad (1)$$

Where E is the total energy of the incident nucleons in the center of gravity system and M is the nucleon mass. The uncontracted volume of the sphere is taken as:

$$\Omega_0 = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} \left( \frac{\hbar}{Mc} \right)^3 \quad (2)$$

The choice of interaction volume is clearly arbitrary and can be adjusted to improve agreement between theory and experiment. The size of the chosen volume radically affects the statistical weight of the process in this model.

There are several complications which arise from the fact that the particles are not independent.

1. In the center of mass system, position and momentum co-ordinates of only n-1 of the n particles are independent variables so that the exponent of the interaction volume should be n-1. Also in momentum space, the volume

corresponding to the total energy should be  $3n-3$  dimensional rather than  $3n$  dimensional.

2. Some of the particles may be identical; that is, indistinguishable and account should be taken of this in the calculation of the volume of momentum space corresponding to this total energy. For example, the corresponding density of states is reduced by  $n!$
3. Some of the particles may carry a spin. One should allow for the corresponding multiplicity of states. If the angular momentum is high, this effect may be small.
4. If isotopic spin is considered as a good quantum number then this quantity should be conserved.
5. One must conserve the differences of the number of nucleons and anti-nucleons.
6. Angular momentum can be included in this model. Its insertion restricts the statistical equilibrium to states with angular momentum equal to that of the two colliding nucleons. Except in extremely relativistic cases, Fermi claims that results will differ only by a small numerical factor.

#### REMARKS ABOUT THE FERMI MODEL

Consider  $n$  particles in mass  $M$  in the calculation of the volume of phase space, one has:

$$\int \prod_i d^3 x_i d^3 p_i \quad (3)$$

1. Fermi assumed that the interaction volume is contracted by the ad hoc factor of  $\frac{2M}{E}$ . The origin of this factor is not clear.

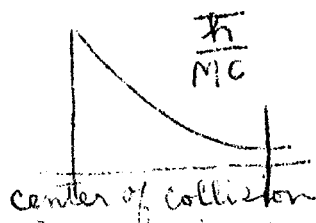
2. Suppose that these are  $n$  classical particles confined to a fixed volume  $\Omega$ . Classically one cannot consider conservation of momentum in this finite volume without considering the interaction of the particles with the volume. It is incorrect to say that the result of the spatial integration is  $\Omega^{n-1}$ . The correct answer depends on the masses of the particles concerned.

Such spatial correlation factors are not large in the non-relativistic case but are very important in the relativistic case. These correlations would be taken into account automatically in the field theory approach by the wave functions.

#### NEUMAN'S METHOD

An approach to the problem of assignment of relative a priori probabilities to the final states which is consistent with translational rotational and Lorentz invariant properties of a colliding system has been developed by Maurice Neuman.

Assume that at the instant of collision, a statistical equilibrium is established. At the center of collision assume that a cloud of virtual particles is formed. Now try to answer the question as to what confines these particles. As an example, consider a non-relativistic particle mass  $M$ . If this particle is to be a virtual particle, then its density versus its position will be something like this:



(4)

The phase volume associated with the virtual particles will depend on the particles' mass. For example, phase volume for virtual pions will be larger than that for heavy mesons which in turn will be larger than that for nucleons. This view is suggested by the uncertainty principle. To insure well defined probabilities, one may insert a Gaussian factor  $e^{-\left(\frac{Mc}{\hbar}\right)^2 x^2}$  in the spatial integrals for each particle.

The non-relativistic case restricted in accord with momentum and energy conservation is therefore:

$$\prod_i d^3 x_i d^3 p_i \delta(\sum_i p_i) \delta(E - \sum_i E_i) e^{-\frac{\nu_i^2 c^2 x_i^2}{\hbar^2}} \delta(\sum_i \nu_i x_i / \sum_i \nu_i) \quad (5)$$

Now consider the same situation relativistically. In this case conservation of center of gravity becomes conservation of center of energy: for example, for a single particle:

$$L_{4X} = (t \vec{p} - \vec{X} E) \quad (6)$$

for a collection of particles then:

$$\sum_i L_{4X} = \sum_i (t \vec{p}_i - E_i \vec{X}_i) = 0 \quad (7)$$

In a world system:

$$\sum_i t \vec{p}_i = 0 \quad (8)$$

Thus, for several particles.

$$\sum_i x_i E_i \quad \text{is conserved} \quad (9)$$

The center of energy is then:

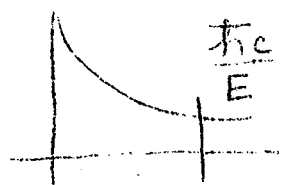
$$\vec{X} = \frac{\sum_i \vec{X}_i E_i}{E} \rightarrow \frac{\sum_i \vec{X}_i m_i}{\sum_i m_i} \quad (10)$$

*in the non-relativistic limit this becomes*

Inserting this 3-dimensional  $\delta$  function to conserve the center of energy, one then has for the phase integral:

$$\prod_i d^3 \vec{x}_i d^3 \vec{p}_i \delta(\sum_i \vec{p}_i) \delta(E - \sum_i E_i) \delta\left(\frac{\sum_i \vec{x}_i E_i}{E}\right) \quad (11)$$

The range of virtual particles relativistically speaking is determined by their energy

$$\frac{\hbar c}{E} \quad \text{instead of} \quad \frac{\hbar}{Mc} \quad (12)$$


One should therefore insert factors  $e^{-\frac{E_i}{\hbar^2 c^2} x_i^2}$  for each particle.  
 After the spatial integrals are performed, one finds:

$$S \sim \frac{E^3}{n^{\frac{3}{2}}} \int \prod_i \frac{d^3 \vec{p}_i}{(\vec{p}_i^2 + M_i^2)^{\frac{3}{2}}} \delta(E - \sum_i (\vec{p}_i^2 + M_i^2)) \delta(\sum_i \vec{p}_i) \quad (13)$$

As a consequence of the conservation of center of energy the volume associated with high momentum is severely restricted. Calculations based on this model are being carried out by Stewart, Neuman, and Lepore, and are still in process.

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